Transformations

In geometry we use input/output process when we determine how shapes are altered or moved. Geometric objects can be moved in the coordinate plane using a coordinate rule. These rules can alter the shape in many different ways. Some rules will translate the shape, some will rotate or reflect the shape, some will stretch or distort the shape, some will increase the size of the shape, etc… lots of different things can happen. Look at the example below:

\[
\begin{align*}
\text{Input} & \quad \text{Output} \\
T(x, y) & \quad (x + 3, y - 6) \\
\text{Pre-Image} & \quad \text{Image}
\end{align*}
\]

So rule T will move or map all points (x, y) by adding 3 to the x value of each point, and subtracting 6 from each y value of each point.

The name of the rule is a capital letter before the (x, y). The point (x, y) represents that all points in the plane will be effected by this rule. The arrow represents the geometric term of map or mapping, which is a geometric way of saying moves. Finally the coordinate description after the mapping arrow symbol represents the change produced by the rule. Can you see the connection to functions? You will input values into the coordinate rule and it will output new values, just as function did. A change is that we refer to the input value as the pre-image, the original location of the point, and the output value as the image, the new location of the point. To help clarify the difference between a pre-image and an image, we use special notation. If the pre-image is point A, then its image will be A' (said A prime). The prime notation tells us that it is an image.

<table>
<thead>
<tr>
<th>1. Given coordinate rule</th>
<th>2. Given coordinate rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(x, y) (\rightarrow) (x + 3, y – 6) determine the</td>
<td>G(x, y) (\rightarrow) (2x + 3, y + 1) determine the</td>
</tr>
<tr>
<td>pre-image of A(-1, 5)?</td>
<td>image of B(5, 2)?</td>
</tr>
<tr>
<td>Pre-image T(x, y) (\rightarrow) (x + 3, y – 6)</td>
<td>Pre-image G(x, y) (\rightarrow) (2x, y + 1)</td>
</tr>
<tr>
<td>T(-1, 5) (\rightarrow) (-1 + 3, 5 – 6)</td>
<td>G(5, 2) (\rightarrow) (2(5), 2 + 1)</td>
</tr>
<tr>
<td>T(-1, 5) (\rightarrow) (2, -1)</td>
<td>G(5, 2) (\rightarrow) (10, 3)</td>
</tr>
<tr>
<td>Pre-image A(-1, 5)</td>
<td>Pre-image B(5, 2)</td>
</tr>
</tbody>
</table>

| Image A'(2, -1) | Image B'(10, 3) |

Working backwards is performing the inverse operation. + ↔ - and x ↔ \(\div\)

<table>
<thead>
<tr>
<th>3. Given coordinate rule</th>
<th>4. Given coordinate rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(x, y) (\rightarrow) (x + 3, y – 6) determine the</td>
<td>T(x, y) (\rightarrow) (2x, y + 1) determine the</td>
</tr>
<tr>
<td>the pre-image of C(3, 2)?</td>
<td>the pre-image of D'(-4, 11)?</td>
</tr>
<tr>
<td>Pre-image T(x, y)</td>
<td>Pre-image T(x, y)</td>
</tr>
<tr>
<td>3 = x + 3</td>
<td>2x = -4</td>
</tr>
<tr>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>x = 0</td>
<td>(x = -2)</td>
</tr>
<tr>
<td>Pre-image C(0, 8)</td>
<td>Pre-image D(-2, 10)</td>
</tr>
</tbody>
</table>

| Image C'(3, 2) | Image D\'(-4, 11) |
Mapping -- The word mapping is used in geometry as the word function is used in algebra. While a mapping is a correspondence between sets of points, a function is a correspondence between sets of numbers. **Function and Mapping mean the same thing but just in two different contexts, in Algebra and in Geometry.** You can not repeat the first value (x), but you can repeat the second value (y).

A correspondence between the pre-image and image is a **MAPPING IF AND ONLY IF** each member of the pre-image corresponds to one and only one member of the image.

In geometry, when you same the same number of points in the pre-image as in the image, it is called a **TRANSFORMATION** A transformation guarantees that if our pre-image has three points, then our image will also have three points.

An **ISOMETRIC TRANSFORMATION (RIGID MOTION)** is a transformation that preserves the distances and/or angles between the pre-image and image.
A NON-ISOMETRIC TRANSFORMATION (NON-RIGID MOTION) is a transformation that does not preserve the distances and angles between the pre-image and image.

1. Use coordinate rule, \( T(x, y) \rightarrow (x + 5, y - 1) \) to determine the missing coordinates.

   \[
   \begin{align*}
   A (4, 5) & \rightarrow A'(\text{____, ____}) \\
   (x + 5, y - 1) & \rightarrow (1, 4) \\
   (-4 + 5, 5 - 1) & = (1, 4)
   \end{align*}
   \]

2. Use coordinate rule, \( G(x, y) \rightarrow (x^2, 2y) \) to determine the missing coordinates.

   \[
   \begin{align*}
   a) A (-4, 5) & \rightarrow A'(\text{____, ____}) \\
   (-4)^2 & = 16, 2(10) \\
   A' & = (16, 20) \\
   b) A (-4, 5) & \rightarrow A'(\text{____, ____}) \\
   (-4)^2 & = 16, 2(10) \\
   A' & = (16, 20)
   \end{align*}
   \]

7. Using the function machine to determine the missing output or input values.

   \[
   \begin{align*}
   \text{Output} & = _____ \\
   \text{Output} & = _____ \\
   \text{Output} & = -47 \\
   \text{Output} & = 21
   \end{align*}
   \]

   \[
   \begin{align*}
   \text{a) Input} & = -6 \\
   4x - 3 & = -24 \\
   4(\frac{-2}{5}) & = - \frac{8}{5} + \frac{15}{5} \\
   x & = \frac{-13}{5}
   \end{align*}
   \]

   \[
   \begin{align*}
   \text{b) Input} & = \frac{2}{5} \\
   4x - 3 & = -47 = 4x - 3 \\
   4(\frac{2}{5}) & = -8/5 + 15/5 \\
   x & = -11
   \end{align*}
   \]

   \[
   \begin{align*}
   \text{c) Input} & = _____ \\
   4x - 3 & = 21 = 4x - 3 \\
   4(\frac{2}{5}) & = -13/5 + 3 + 3 + 3 + 3 \\
   x & = 6
   \end{align*}
   \]

   \[
   \begin{align*}
   \text{d) Input} & = _____ \\
   4x - 3 & = 21 = 4x - 3 \\
   4(\frac{2}{5}) & = -8/5 + 15/5 \\
   x & = -11
   \end{align*}
   \]

8. Determine the rule of the function machine.

   \[
   \begin{array}{|c|c|c|c|c|}
   \hline
   \text{First Input} & \text{Second Input} & \text{Third Input} & \text{Fourth Input} & \text{What is the rule for this function machine?} \\
   \hline
   \text{INPUT} & \text{OUTPUT} & \text{INPUT} & \text{OUTPUT} & \text{INPUT} & \text{OUTPUT} & \text{INPUT} & \text{OUTPUT} \\
   \hline
   \hline
   0 & \rightarrow -1 & 1 & \rightarrow 1 & 2 & \rightarrow 3 & 2 & \rightarrow 5 & 2 \\
   \hline
   \end{array}
   \]

7. Determine three different function machine rules for the given input/output.
8. Use the given coordinate rules to solve missing coordinates.

a) \( T (x, y) \rightarrow (x, y + 7) \)

\[ A (-4,9) \quad A' (-4,16) \]

\[ B (5,-7) \quad B' (5,0) \]

b) \( S (x, y) \rightarrow (-y, x) \)

\[ A (-4,9) \quad A' (-9,-4) \]

\[ B (7,-9) \quad B' (9,7) \]

c) \( F (x, y) \rightarrow (5x, 3y) \)

\[ A (-4,9) \quad A' (-20,27) \]

\[ B (-1,4) \quad B' (-5,12) \]

d) \( G (x, y) \rightarrow (-x, -3y) \)

\[ A (-4,9) \quad A' (4,12) \]

\[ B (8,-y) \quad B' (-8,-24) \]

e) \( H (x, y) \rightarrow (2x - 1, y - 3) \)

\[ A (-4,9) \quad A' (-9,6) \]

\[ B (16,186) \quad B' (31,15) \]

f) \( P (x, y) \rightarrow (x + 3, 2y) \)

\[ A (-4,9) \quad A' (-1,18) \]

\[ B (2,4) \quad B' (5,8) \]

9. Write out the English translation for the following coordinate rule.

\[ F (x, y) \rightarrow (x - 4, 2y) \]

Coordinate Rule: Subtract 4 from \( x \) and double \( y \) value

10. Complete the Analogy. Input IS TO Output AS Pre-Image IS TO Image.

1. Given \( f(x) = x^2 \)

a) find \( f(-2) = (-2)^2 = 4 \)

b) find \( f(2) = 2^2 = 4 \)

c) \( f(0) = (0)^2 = 0 \)

d) \( f(5) = (5)^2 = 25 \)

e) Use the data in 1a-d to complete the diagram to the right.

f) is \( f(x) = x^2 \) a function? YES OR NO

g) is \( f(x) = x^2 \) a one to one function? YES OR NO

2. Given \( f(x) = -5x + 6 \)
3. Given \( f(x) = |x-1| \)

a) find \( f(-2) = |-2-1| = 3 \)  
b) find \( f(2) = |2-1| = 1 \)  
c) find \( f(0) = |0-1| = 1 \)  
d) find \( f(5) = |5-1| = 4 \)

e) Use the data in 3a-d to complete the diagram to the right.

f) is \( f(x) = |x-1| \) a function? YES OR NO  
g) is \( f(x) = |x-1| \) a one to one function? YES OR NO

4. Determine whether the following are functions or not.

4. Set A \( \rightarrow \) Set B  
Function? YES OR NO

5. Set A \( \rightarrow \) Set B  
Function? YES OR NO

6. Set A \( \rightarrow \) Set B  
Function? YES OR NO

7. Set A \( \rightarrow \) Set B  
Function? YES OR NO

8. Set A \( \rightarrow \) Set B  
Function? YES OR NO

9. Set A \( \rightarrow \) Set B  
Function? YES OR NO

10. - 15. Given that the pre-image is Quadrilateral ABCD, determine which of the following could be classified as a mapping of the plane.

10. Function? YES OR NO  

11. Function? YES OR NO

12. Function? YES OR NO
13. Which of the problems in question 5, would be classified as TRANSFORMATIONS?

10, 14, 15

14. Transformations are a specific type of mapping. What makes them special from the general process of mapping?

Each point in the pre-image will have a distinct point in the image (one to one)

15. Jeff is given a question on a test about transformations. He is given two examples both with pre-image hexagon ABCDE. The question asks if the two shapes are a transformation or not. On the first one he said Yes they are transformations because they are identical but in a different location and on the second one he said No that it was not a transformation because they were different shapes. Is he correct? Explain why you agree or disagree?

Example 1

An isometric transformation (reflection) (one to one)

Example 2

A non-isometric Transformations (one to one)

Line of Reflections

The line of reflection is the perpendicular bisector of the segment joining every point and its image.
Remember that a reflection is a flip. Under a reflection, the figure does not change size.
A line reflection creates a figure that is congruent to the original figure and is called an isometry (a transformation that preserves length). Since naming (lettering) the figure in a reflection requires changing the order of the letters (such as from clockwise to counterclockwise), a reflection is more specifically called a non-direct or opposite isometry.

Under a Line reflection, distance, angle measurement, collinearity and midpoint are preserved

### Line Reflections in the Coordinate Plane

<table>
<thead>
<tr>
<th>Reflection in the y-axis $\rightarrow$ $P(x, y)$ is $P'(\text{\textminus}x, y)$</th>
<th>Reflection in the x-axis $\rightarrow$ $P(x, y)$ is $P'(x, -y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{y-axis}(x, y) = (\text{-}x, y)$</td>
<td>$r_{x-axis}(x, y) = (x, -y)$</td>
</tr>
</tbody>
</table>

**Reflection in the y-axis**

- Point $C'$ is 5 units to the left of the y-axis.
- Point $B'$ is 4 units above the x-axis.

**Reflection in the x-axis**

- Point $C$ is 5 units to the right of the y-axis.
- Point $B$ is 4 units above the x-axis.

**Reflection in the line $y = x$**

$P(x, y)$ is $P'(y, x)$

**Reflection over any Line $r_{x} = \cdot 2(x, y)$**
1. Under a reflection in the x-axis, the image of \((x, y)\) is \((x, -y)\).

2. Under a reflection in the y-axis, the image of \((x, y)\) is \((-x, y)\).

3. Under a reflection in the line \(y = x\), the image of \((x, y)\) is \((y, x)\).

Find the image of the point under a reflection in the x-axis

<table>
<thead>
<tr>
<th>Point</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5, 7))</td>
<td>((-5, 7))</td>
</tr>
<tr>
<td>((-5, 7))</td>
<td>((5, 7))</td>
</tr>
<tr>
<td>((3, 0))</td>
<td>((-3, 0))</td>
</tr>
</tbody>
</table>

Find the image of the point under a reflection in the y-axis

<table>
<thead>
<tr>
<th>Point</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5, 7))</td>
<td>((-5, 7))</td>
</tr>
<tr>
<td>((-5, 7))</td>
<td>((5, 7))</td>
</tr>
<tr>
<td>((-1, -6))</td>
<td>((1, -6))</td>
</tr>
</tbody>
</table>

Find the image of the point under a reflection in the line \(y = x\)

<table>
<thead>
<tr>
<th>Point</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5, 7))</td>
<td>((7, 5))</td>
</tr>
<tr>
<td>((7, 5))</td>
<td>((5, 7))</td>
</tr>
<tr>
<td>((6, 6))</td>
<td>((6, 6))</td>
</tr>
</tbody>
</table>
16. The vertices of \( \triangle ABC \) are A(3, 0), B(3, 6) and C(0, 6). Find the coordinates of the images of the vertices under the given reflection:

Given
the x-axis \hspace{1cm} \text{the y-axis} \hspace{1cm} \text{the line } y = x
A(3, 0) \hspace{1cm} (3, 0) \hspace{1cm} (-3, 0) \hspace{1cm} (0, 3)
B(3, 6) \hspace{1cm} (3, -6) \hspace{1cm} (-3, 6) \hspace{1cm} (6, 3)
C(0, 6) \hspace{1cm} (0, -6) \hspace{1cm} (0, 6) \hspace{1cm} (6, 0)

17. The vertices of \( \triangle ABC \) are A(1, 4), B(3, 0) and C(-3, -4). Find the coordinates of the images of the vertices under the given reflection:

Given
the x-axis \hspace{1cm} \text{the y-axis} \hspace{1cm} \text{the line } y = x
A(1, 4) \hspace{1cm} (1, -4) \hspace{1cm} (-1, 4) \hspace{1cm} (4, 1)
B(3, 0) \hspace{1cm} (3, 0) \hspace{1cm} (-3, 0) \hspace{1cm} (0, 3)
c(-3, -4) \hspace{1cm} (-3, 4) \hspace{1cm} (-3, 4) \hspace{1cm} (-4, -3)

Using the given coordinates, draw \( \triangle ABC \) and \( \triangle A'B'C' \) on one set of axes and then find the equation of the line of reflection.

18. \( \triangle ABC \): A(2, 4), B(2, 1) and C(-1, 1)
\( \triangle A'B'C' \): A'(4, 4), B'(4, 1) and C'(7, 1)

19. \( \triangle ABC \): A(1, 3), B(2, 5) and C(5, 3)
\( \triangle A'B'C' \): A'(1, 1), B'(2, -1) and C'(5, 1)
20. ΔABC: A(1, 4), B(2, 1) and C(4, 2)  
ΔA’B’C”: A’(-5, 4), B’(-6, 1) and C’(-8, 2)

21. ΔABC: A(4, 2), B(6, 2) and C(2, -1)  
ΔA’B’C”: A’(2, 4), B’(2, 6) and C’(-1, 2)

22. ΔABC: A(-1, 0), B(0, 2) and C(4, 1)  
ΔA’B’C”: A’(-1, 8), B’(0, 6) and C’(4, 7)

23. ΔABC: A(2, -1), B(4, 2) and C(-1, 2)  
ΔA’B’C”: A’(1, -2), B’(-2, -4) and C’(-2, 1)
(1) REFLECTION OVER THE Y AXIS

<table>
<thead>
<tr>
<th>Pre-Image Location</th>
<th>Reflection</th>
<th>Image Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Place ΔDEF on the coordinate grid at D(0,6), E(3,6) and F(0,2)</td>
<td>( R_{y \text{ axis}} )</td>
<td>D’(0,6), E’(-3,6), &amp; F’(0,2)</td>
</tr>
<tr>
<td>b) Place ΔABC on the coordinate grid at A(3,0), B(5,-5) and C(1,-5)</td>
<td>( R_{y \text{ axis}} )</td>
<td>A’(-3,0), B’(-5,-5), &amp; C’(-1,-5)</td>
</tr>
<tr>
<td>c) Place ΔMNP on the coordinate grid at M(0,-1), N(7,-4) and P(2,-4)</td>
<td>( R_{y \text{ axis}} )</td>
<td>M’(0,-1), N’(-7,-4), &amp; P’(-2,-4)</td>
</tr>
</tbody>
</table>

Create a transformation rule for reflection over the y axis. \( R_{y \text{ axis}} (x, y) = (-x, y) \)

(2) REFLECTION OVER THE X AXIS

<table>
<thead>
<tr>
<th>Pre-Image Location</th>
<th>Reflection</th>
<th>Image Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Place ΔDEF on the coordinate grid at D(-7,-2), E(-4,-2) and F(-7,-6)</td>
<td>( R_{x \text{ axis}} )</td>
<td>D’(-7,2), E’(-4,2), &amp; F’(-7,6)</td>
</tr>
<tr>
<td>b) Place ΔABC on the coordinate grid at A(6,-1), B(8,-6) and C(4,-6)</td>
<td>( R_{x \text{ axis}} )</td>
<td>A’(6,1), B’(8,6), &amp; C(4,6)</td>
</tr>
<tr>
<td>c) Place ΔMNP on the coordinate grid at M(-1,-2), N(6,-5) and P(1,-5)</td>
<td>( R_{x \text{ axis}} )</td>
<td>M’(-1,2), N’(6,5), &amp; P’(1,5)</td>
</tr>
</tbody>
</table>

Create a transformation rule for reflection over the x axis. \( R_{x \text{ axis}} (x, y) = (x, -y) \)

(3) REFLECTION OVER THE Y = 1X LINE

<table>
<thead>
<tr>
<th>Pre-Image Location</th>
<th>Reflection</th>
<th>Image Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Place ΔDEF on the coordinate grid at D(0,6), E(3,6) and F(0,2)</td>
<td>( R_{y = x} )</td>
<td>D’(6,0), E’(6,3), &amp; F’(2,0)</td>
</tr>
<tr>
<td>b) Place ΔABC on the coordinate grid at A(3,0), B(5,-5) and C(1,-5)</td>
<td>( R_{y = x} )</td>
<td>A’(0,3), B’(-5,5), &amp; C’(-5,1)</td>
</tr>
<tr>
<td>c) Place ΔMNP on the coordinate grid at M(0,-1), N(7,-4) and P(2,-4)</td>
<td>( R_{y = x} )</td>
<td>M’(-1,0), N’(-4,7), &amp; P’(-4,2)</td>
</tr>
</tbody>
</table>

Create a transformation rule for reflection over the y = x line. \( R_{y = x} (x, y) = (y, x) \)

(4) REFLECTION OVER A VERTICAL LINE (x = c) (c is the variable representing all possible vertical lines)

<table>
<thead>
<tr>
<th>Pre-Image Location</th>
<th>Reflection</th>
<th>Image Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Place ΔDEF on the coordinate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
grid at D(-4,5), E(-1,5) and F(-4,1) $R_{x=0}$ (y axis) $D'(4,5)$ $E'(1,5)$, & $F'(4,1)$

b) Place $\triangle DEF$ on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1) $R_{x=1}$ $D'(6,5)$ $E'(3,5)$, & $F'(6,1)$

c) Place $\triangle DEF$ on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1) $R_{x=2}$ $D'(8,5)$ $E'(5,5)$, & $F'(8,1)$

d) Place $\triangle DEF$ on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1) $R_{x=3}$ $D'(10,5)$ $E'(7,5)$ & $F'(10,1)$

e) Place $\triangle DEF$ on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1) $R_{x=4}$ $D'(12,5)$ $E'(9,5)$ & $F'(12,1)$

1. – 10. Use the to reflect the following figures over their respective line m. Label the image.

<table>
<thead>
<tr>
<th>1.</th>
<th>2.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.jpg" alt="Image 1" /></td>
<td><img src="image2.jpg" alt="Image 2" /></td>
</tr>
<tr>
<td>3.</td>
<td>4.</td>
</tr>
</tbody>
</table>
11. – 16. Determine the line of reflection for the following pre-image and images.

11. 

12. 

13. 

14. 

15. 

16. 

3. Determine the pre-image coordinates, then reflect it, and determine the image coordinates.

a) A = (-8, -2)    \[R_{x\text{axis}}(A)\]    A' = (-8, 2)

b) B = (-4, 2)    \[R_{y\text{axis}}(B)\]    B' = (4, 2)

c) C = (-6, 7)    \[R_m(C)\]    C' = (2, 7)

d) D = (4, 4)    \[R_{x\text{axis}}(D)\]    D' = (4, -4)

e) E = (2, 0)    \[R_{x\text{axis}}(E)\]    E' = (2, 0)

f) F = (8, -5)    \[R_n(F)\]    F' = (8, -7)

g) G = (0, -4)    \[R_{y\text{axis}}(G)\]    G' = (0, -4)

4. Determine the name of the point that meets the given conditions.
5. Determine the name of the point that meets the given conditions.

\[
\begin{align*}
a) \ R_m(A) &= \text{D} \\
b) \ R_{FC}(A) &= \text{E} \\
c) \ R_h(B) &= \text{E} \\
d) \ R_{AD}(A) &= \text{B} \\
e) \ R_n(D) &= \text{E} \\
f) \ R_n(B) &= \text{A} \\
g) \ R_{FC}(D) &= \text{B} \\
h) \ R_{BE}(A) &= \text{C}
\end{align*}
\]

**LINE SYMMETRY (or REFLECTIONAL SYMMETRY)**

A line of symmetry divides a figure into halves that are the **mirror images** of each other.

Below are some examples of figures that do have line symmetry. A shape can have more than one line symmetry.

Here are a few non-examples of shapes DO NOT have line symmetry.
The maximum lines of symmetry that a polygon can have are equal to its number of sides. The maximum is always found in the regular polygon, because all sides and all angles are congruent.

1. Below are the letters of the alphabet. Draw one or more lines of reflections for each of these letters, if they have one.

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td></td>
</tr>
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<td></td>
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<td></td>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>K</td>
<td>L</td>
<td></td>
<td>N</td>
<td>P</td>
<td>Q</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>T</td>
<td>U</td>
<td>V</td>
<td>W</td>
<td></td>
<td>Y</td>
<td>Z</td>
<td></td>
</tr>
</tbody>
</table>

2. Tell which of the following words have line symmetry and if such symmetry exists, draw the line.

<table>
<thead>
<tr>
<th>DAD</th>
<th>YOUTH</th>
<th>HIKED</th>
<th>CHECK</th>
<th>BOB</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEED</td>
<td>RADAR</td>
<td>CHOKE</td>
<td>AVA</td>
<td>TOT</td>
</tr>
<tr>
<td>AXION</td>
<td>MOM</td>
<td>CHO</td>
<td>HOE</td>
<td>DOB</td>
</tr>
</tbody>
</table>

3. Draw all lines of symmetry in the geometric shapes, if there are none, leave alone:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>square</td>
<td>rectangle</td>
<td>eclipse</td>
<td>circle</td>
</tr>
</tbody>
</table>
Point Reflection

A **Point Reflection** creates a figure that is congruent to the original figure and is called an **isometry** (a transformation that preserves length). Since the orientation in a point reflection remains the same (such as counterclockwise seen in this diagram), a point reflection is more specifically called a **direct isometry**. P is the midpoint of each line AA', BB' CC'.
Under a Point of Reflection, distance, angle measurement, collinearity and midpoint are preserved.

**Point of Symmetry**

In a point symmetry, the center point is a midpoint to every segment formed by joining a point to its image.

**Point Reflection in the Coordinate Plane**

Reflection over the Origin $P(x, y)$ is $P'(-x, -y)$

$r_o (x, y) = (-x, -y)$

Here are some examples of shapes that have rotation symmetry.

<table>
<thead>
<tr>
<th>Example #1</th>
<th>Example #2</th>
<th>Example #3</th>
<th>Example #4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Example 1" /></td>
<td><img src="image2.png" alt="Example 2" /></td>
<td><img src="image3.png" alt="Example 3" /></td>
<td><img src="image4.png" alt="Example 4" /></td>
</tr>
</tbody>
</table>
ANGLE OF ROTATION - When a shape has rotational symmetry we sometimes want to know what the angle of rotational symmetry is. To determine this we determine the SMALLEST angle through which the figure can be rotated to coincide with itself. This number will always be a factor of 360. So in the examples above, example #1 would have angle of rotational symmetry of 120, example #2 would have an angle of 180, example #3 would have an angle of 120° and finally example #4 would have an angle of 180.

ORDER OF A ROTATION SYMMETRY -- The number of positions in which the object looks exactly the same is called the order of the symmetry. When determining order, the last rotation returns the object to its original position. **Order 1 implies no true rotational symmetry since a full 360 degree rotation was needed.** So in the four examples of rotation symmetry, example #1 has order 3, example #2 has order 2, example #3 has order 3, and example #4 has order 2.

Here are a few non-examples of shapes DO NOT have rotation symmetry.

<table>
<thead>
<tr>
<th>NON - Example #1</th>
<th>NON - Example #2</th>
<th>NON - Example #3</th>
<th>NON - Example #4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Example #1" /></td>
<td><img src="image2.png" alt="Example #2" /></td>
<td><img src="image3.png" alt="Example #3" /></td>
<td><img src="image4.png" alt="Example #4" /></td>
</tr>
</tbody>
</table>

(the colors don’t work) (numbers don’t work)

1. Draw a point of symmetry for each letter that has one.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>K</td>
<td>L</td>
<td>M</td>
<td>N</td>
<td>O</td>
<td>P</td>
<td>Q</td>
<td>R</td>
</tr>
<tr>
<td>S</td>
<td>T</td>
<td>U</td>
<td>V</td>
<td>W</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td></td>
</tr>
</tbody>
</table>

2. Draw a point of symmetry if the word has one.

<table>
<thead>
<tr>
<th>Sis</th>
<th>Wow</th>
<th>Noon</th>
<th>Zoo</th>
<th>Xox</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un</td>
<td>Pod</td>
<td>Choked</td>
<td>Swims</td>
<td>Ox</td>
</tr>
</tbody>
</table>

3. Draw points of symmetry in the geometric shapes, and give the angle of rotation and the
<table>
<thead>
<tr>
<th>order of rotation.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="square" /></td>
</tr>
<tr>
<td>equilateral triangle</td>
</tr>
<tr>
<td><img src="image" alt="rhombus" /></td>
</tr>
<tr>
<td>pentagon</td>
</tr>
</tbody>
</table>

3. Under a reflection in the origin, the image of (x, y) is \((-x, -y)\)

4. Using the reflection through the origin, find the images of A(2, 1), B(4, 5) and C(-1, 3) and draw \(\triangle ABC\) and \(\triangle A'B'C'\) on the set of axis.
   - \(A(2, 1) \rightarrow A'(2, -1)\)
   - \(B(4, 5) \rightarrow B'(-4, -5)\)
   - \(C(-1, 5) \rightarrow C'(1, 5)\)

5. Using the reflection through the origin, find the images of A(-5, -2), B(-1, -2) and C(-1, 4), and D(-3, 3) and draw \(\text{ABCD}\) and \(A'B'C'D'\) on the set of axis.
   - \(A(-5, -2) \rightarrow A'(5, 2)\)
   - \(B(-1, -2) \rightarrow B'(1, 2)\)
   - \(C(-1, 4) \rightarrow C'(1, -4)\)
   - \(D(-3, 3) \rightarrow D'(3, -3)\)
The image of $\triangle ABC$ under a point of reflection is $\triangle A'B'C'$. Using the given coordinates, draw $\triangle ABC$ and $\triangle A'B'C'$ on one set of axis, then find the coordinates of the point of symmetry.

6. $\triangle ABC$: A(2, 2), B(5, 2) and C(5, 4)  
   $\triangle A'B'C'$: A'(0, 2), B'(-3, 2) and C'(-3, 0)  
   Point of Symmetry (1, 2)

7. $\triangle ABC$: A(2, 5), B(5, 6) and C(2, 3)  
   $\triangle A'B'C'$: A'(2, 1), B'(-1, 0) and C'(2, 3)  
   Point of Symmetry (2, 3)

8. $\triangle ABC$: A(3, 1), B(5, 5) and C(6, 3)  
   $\triangle A'B'C'$: A'(3, 7), B'(-1, 3) and C'(0, 5)  
   Point of Symmetry (3, 4)

9. $\triangle ABC$: A(-3, 6), B(1, 7) and C(2, 4)  
   $\triangle A'B'C'$: A'(-1, 0), B'(-5, -1) and C'(-6, 2)  
   Point of Symmetry (-2, 3)
10. \( \triangle ABC: \ A(-1, 1), B(2, 5) \) and \( C(5, 1) \)
\( \triangle A'B'C': \ A'(3, 3), B'(0, -1) \) and \( C'(-3, 3) \)
Point of Symmetry (1, 2)

11. \( \triangle ABC: \ A(3, 8), B(0, 2) \) and \( C(5, 4) \)
\( \triangle A'B'C': \ A'(1, 4), B'(4, 10) \) and \( C'(-1, 8) \)
Point of Symmetry (2, 6)

12. The vertices of \( \triangle DEF \) are \( D(4, 3), E(8, 1) \) and \( F(8, 3) \). If \( \triangle DEF \) is reflected through the point (4, 1), find the coordinates of the images \( \triangle D'E'F' \)

13. The vertices of \( \triangle RST \) are \( R(0, 0), S(4, -4) \) and \( T(5, -1) \). If \( \triangle RST \) is reflected through the point (2, 1), find the coordinates of the images \( \triangle R'S'T' \)
14. The vertices of $\triangle ABC$ are $A(1, 3)$, $B(1, 1)$ and $C(5, 1)$.
   a) give the points of $\triangle A'B'C'$ under a point reflection of $\triangle ABC$ through the origin.
   b) give the points of $A''B''C''$, under a reflection of $\triangle A'B'C'$ in the x-axis

15. The vertices of $\triangle ABC$ are $A(1, 3)$, $B(1, 1)$ and $C(5, 1)$.
   a) give the points of $\triangle A'B'C'$ under a reflection of $\triangle ABC$ through the equation $x = 1$.
   b) give the points of $A''B''C''$, under a reflection of $\triangle A'B'C'$ in the y-axis
Translations

A translation "slides" an object a fixed distance in a given direction. The original object and its translation have the same shape and size, and they face in the same direction. A translation creates a figure that is congruent with the original figure and preserves distance (length) and orientation (lettering order). A translation is a direct isometry. The word "translate" in Latin means "carried across".

Under a Translation, distance, angle measurement, collinearity and midpoint are preserved.

Translactions in the Coordinate Plane

$T_{a,b}(x, y) = (x+a, y + b)$

The translation below, moves the figure 7 units to the left and 3 units down.

$T_{7,-3}(x, y) = (x+7, y-3)$
Use the rule \((x, y) \rightarrow (x + 2, y + 3)\) to find the image of the given point:

1. \((3, -1)\) \rightarrow \((5, 2)\)  
2. \((3, -1)\) \rightarrow \((5, 2)\)  
3. \((3, -1)\) \rightarrow \((5, 2)\)  
4. \((3, -1)\) \rightarrow \((5, 2)\)

Use the rule \((x, y) \rightarrow (x - 4, y + 9)\) to find the image of the given point:

5. \((4, 4)\) \rightarrow \((0, 13)\)  
6. \((4, 4)\) \rightarrow \((0, 13)\)  
7. \((4, 4)\) \rightarrow \((0, 13)\)  
8. \((4, 4)\) \rightarrow \((0, 13)\)

Find the image of the point \((2, 7)\) under the given translation:

9. \(T_{1, 2}\) \((2, 7)\) \rightarrow \((3, 9)\)  
10. \(T_{1, 2}\) \((2, 7)\) \rightarrow \((3, 9)\)  
11. \(T_{1, 2}\) \((2, 7)\) \rightarrow \((3, 9)\)  
12. \(T_{1, 2}\) \((2, 7)\) \rightarrow \((3, 9)\)

Find the rule for the translation so the image of A is A`

13. \(A(3, 8) \rightarrow A'(4, 6)\):
   \(x + 1 = 4\) and \(y - 2 = 6\)
   \((x + 1, y - 2)\) or \(T_{1, 2}\)

14. \(A(-1, 2) \rightarrow A'(-2, -3)\):
   \(-1 + -1 = -2\) and \(2 + -5 = -3\)
   \((x - 1, y - 5)\) or \(T_{-1,-5}\)

15. \(A(3, 8) \rightarrow A'(4, 6)\):
   \(x + 1 = 4\) and \(y - 2 = 6\)
   \((x + 1, y - 2)\) or \(T_{1, 2}\)

16. \(A(-1, 2) \rightarrow A'(-2, -3)\):
   \(-1 + -1 = -2\) and \(2 + -5 = -3\)
   \((x - 1, y - 5)\) or \(T_{-1,-5}\)

19. Given \(\triangle HOT\) whose vertices are H(-2, 0), O(0, 0) and T(0, 4):
   a) Give the coordinates of \(\triangle H'O'T'\) under the image \(T_{2,-3}\) of \(\triangle HOT\)
   b) Give the coordinates of \(\triangle H''O''T''\) under the image of \(T_{1,2}\) of \(\triangle H'O'T'\)
   c) Name a single transformation that is equivalent to \(T_{2,-3}\) followed by \(T_{1,2}\)

20. Given quadrilateral WARM whose vertices are W(-4, 3), A(0, 1), R(2, 5) and M(0, 5):
   a) Give the coordinates of \(\triangle W'A'R'M'\) under the image \(T_{5,-2}\) of WARM
   b) Give the coordinates of \(\triangle W''A''R''M''\) under the image of \(T_{0,2}\) of \(\triangle W'A'R'M'\)
   c) Name a single transformation that is equivalent to \(T_{5,-2}\) followed by \(T_{0,2}\)
1. Use the grid or patty paper to translate the following figures. Label the image.
2. Determine the translation coordinate rule from the vector.

   a) \( T(x, y) \longrightarrow (x - 2, y - 2) \)
   b) \( T(x, y) \longrightarrow (x - 3, y) \)
   c) \( T(x, y) \longrightarrow (x + 3, y - 1) \)

3. Determine the translation rule from the pre-image and image.

   a) \( A(3,5) \) \( A'(1,1) \) \( T(x,y) \longrightarrow (x - 4, y - 2) \)
   b) \( A(-4,11) \) \( A'(3,0) \) \( T(x,y) \longrightarrow (x + 7, y - 11) \)
   c) \( A(0,-8) \) \( A'(-1,-3) \) \( T(x,y) \longrightarrow (x - 1, y + 5) \)
   d) \( A(8,3) \) \( A'(11,3) \) \( T(x,y) \longrightarrow (x + 3, y) \)

4. Given a translation rule, determine the missing point.

   a) \( T(x,y) \longrightarrow (x + 3, y - 5) \) \( A(-4,7) \) \( A'(-1,2) \)
   b) \( T(x,y) \longrightarrow (x - 7, y - 1) \) \( A(9,1) \) \( A'(2,0) \)
   c) \( T(x,y) \longrightarrow (x + 1, y + 6) \) \( A(3,-7) \) \( A'(4,-1) \)
   d) \( T(x,y) \longrightarrow (x, y + 4) \) \( A(8,-4) \) \( A'(8,0) \)
   e) \( T(x,y) \longrightarrow (x + 3, y + 1) \) \( A(-7,0) \) \( A'(-4,1) \)
   f) \( T(x,y) \longrightarrow (x - 8, y - 5) \) \( A(5,2) \) \( A'(-3,-3) \)

5. Convert between vector component form and coordinate form.

   a) \( T_{<5,2>} (A) = \) \( T(x,y) \longrightarrow (x - 5, y + 2) \)
   b) \( T_{<0,-12>} (A) = \) \( T(x,y) \longrightarrow (x, y - 12) \)
c) \( T_{c,-1.5,-7} (A) = \)
\[ T (x,y) \rightarrow (x - 1.5, y - 7) \]

6. Write the coordinate rule that matches the description.
   a) 4 down and 3 right
   \[ T (x,y) \rightarrow (x + 3, y - 4) \]
   b) left 7 and down 2
   \[ T (x,y) \rightarrow (x - 7, y - 2) \]
   c) right 1
   \[ T (x,y) \rightarrow (x + 1, y) \]

7. What is the resultant translation of Point A after mapping T (x,y) followed by R (x,y).
   a) A (4,8)
   \[ T (x,y) \rightarrow (x + 3, y - 7) \]
   \[ A' (1, -1) \]
   \[ R (x,y) \rightarrow (x - 8, y - 2) \]
   \[ A'' (-9, -1) \]
   b) A (2,0)
   \[ T (x,y) \rightarrow (x - 1, y) \]
   \[ A' (1, 0) \]
   \[ R (x,y) \rightarrow (x - 3, y + 3) \]
   \[ A'' (-2, 3) \]
   c) A (5,-11)
   \[ T (x,y) \rightarrow (x + 7, y - 11) \]
   \[ A' (12, -22) \]
   \[ R (x,y) \rightarrow (x - 9, y + 9) \]
   \[ A'' (3, -13) \]

8. Can you find a shortcut to doing two translations?
   \[ T(x, y) \rightarrow (x + 3, y - 5) \]
   \[ R(x, y) \rightarrow (x + 6, y + 8) \]
   \[ (x, y) \rightarrow (x + 3 + 6, y - 5 + 8) \]
   \[ (x, y) \rightarrow (x + 9, y + 3) \] Result is to combine them

9. What is the pre-image of A'(5,4) mapped by translation T (x,y) \rightarrow (x - 5, y + 11)?
   Do the opposite
   \[ A( \quad , \quad ) \] A'(-5, 4)
   \[ x - 5 = -5, y + 11 = 4 \]
   \[ x = 0, y = -7 \]

Glide Reflection

A Glide Reflection is a composition of transformations that consists of a line reflection and a translation in the direction of the line of reflection performed in either order.

Under a Glide Reflection, distance, angle measurement, collinearity and midpoint are preserved. It is an isometry since it preserves distance.

Rotations

A rotation is a transformation that turns a figure about a fixed point called the center of rotation. Rays drawn from the center of rotation to a point and its image form an angle called the angle of rotation. A rotation is an isometry

\[ \text{rotation } R_{\text{degrees}} \]

Under a Rotation, distance, angle measurement, collinearity and midpoint are preserved.
Rotations in the Coordinate Plane

Unless stated, the rotation is always counterclockwise.

<table>
<thead>
<tr>
<th>Rotation of 90°</th>
<th>R_{90}(x, y) = (-y, x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation of 180°</td>
<td>R_{180}(x, y) = (-x, -y)</td>
</tr>
<tr>
<td>Rotation of 270°</td>
<td>R_{270}(x, y) = (y, -x)</td>
</tr>
</tbody>
</table>

Counting the lunch line.

1. Under a rotation of 90° counterclockwise about the origin, the image of (x, y) is \((-y, x)\).

2. Under a rotation of 180° counterclockwise about the origin, the image of (x, y) is \((-x, -y)\).

Using the rule \((x, y) \rightarrow (-y, x)\) and give the coordinates of the image \(\Delta A'B'C'\) and draw \(\Delta ABC\) and \(\Delta A'B'C'\).

3. \(\Delta ABC:\) A(1, 1), B(1, 5) and C(4, 5)

\[\begin{array}{c|c}
(x, y) & (-y, x)R_{90} \\
A(1, 1) & A'(1, -1) \\
B(1, 5) & B'(-5, 1) \\
C(4, 5) & C'(-5, 4) \\
\end{array}\]

4. \(\Delta ABC:\) A(4, 3), B(4, -2) and C(1, -1)

\[\begin{array}{c|c}
(x, y) & (-y, x)R_{90} \\
A(4, 3) & A'(-3, 4) \\
B(4, -2) & B'(2, 4) \\
C(1, -1) & C'(1, 1) \\
\end{array}\]
5. ΔABC: A(1, 2), B(1, 6) and C(3, 2)

\[(x, y) \rightarrow (-y, x)\]  \(R_{90}\)

A(1, 2) → A'(-2, 1)
B(1, 6) → B'(-6, 1)
C(3, 2) → C'(-2, 3)

6. ΔABC: A(1, 3), B(3, 5) and C(3, 0)

\[(x, y) \rightarrow (-y, x)\]  \(R_{90}\)

A(1, 3) → A'(-3, 1)
B(3, 5) → B'(-5, 3)
C(3, 0) → C'(0, 3)

7. ΔABC: A(2, 1), B(3, -1) and C(-2, -1)

\[(x, y) \rightarrow (-y, x)\]  \(R_{90}\)

A(2, 1) → A'(-1, 2)
B(3, -1) → B'(-1, 3)
C(-2, -1) → C'(-1, -2)

8. ΔABC: A(0, 0), B(2, 1) and C(-2, 4)

\[(x, y) \rightarrow (-y, x)\]  \(R_{90}\)

A(0, 0) → A'(0, 0)
B(2, 1) → B'(-1, 2)
C(-2, 4) → C'(-4, -2)
9. $\triangle ABC$: $A(-2, 2), B(0, -4)$ and $C(4, -4)$

$x, y \rightarrow (-y, x)R_{90}$

$A(-2, 2) \rightarrow A'(2, -2)$

$B(0, -4) \rightarrow B'(4, 0)$

$C(4, -4) \rightarrow C'(4, 4)$

10. $\triangle ABC$: $A(2, 5), B(0, 1)$ and $C(-3, 7)$

$x, y \rightarrow (-y, x)R_{90}$

$A(2, 5) \rightarrow A'(-5, 2)$

$B(0, 1) \rightarrow B'(-1, 0)$

$C(-3, 7) \rightarrow C'(7, -3)$

Dilations in the Coordinate Plane
Under a dilation, distance is **not** preserved.

Using the rule \((x, y) \rightarrow (4x, 4y)\), find the image of the given point:

<table>
<thead>
<tr>
<th>Point</th>
<th>New Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ((3, -1))</td>
<td>((12, -4))</td>
</tr>
<tr>
<td>2. ((2, 6))</td>
<td>((8, 24))</td>
</tr>
<tr>
<td>3. ((0, -8))</td>
<td>((0, -32))</td>
</tr>
<tr>
<td>4. ((-5, -3))</td>
<td>((-20, -12))</td>
</tr>
</tbody>
</table>

Find the image of the given point under a dilation of 5

<table>
<thead>
<tr>
<th>Point</th>
<th>New Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. ((5, 7))</td>
<td>((25, 35))</td>
</tr>
<tr>
<td>6. ((6, 2))</td>
<td>((30, 10))</td>
</tr>
<tr>
<td>7. ((-1, -4))</td>
<td>((-5, -20))</td>
</tr>
<tr>
<td>8. ((3, 0))</td>
<td>((15, 0))</td>
</tr>
</tbody>
</table>

Under \(D_{-3}\) find the image of the given point

<table>
<thead>
<tr>
<th>Point</th>
<th>New Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. ((5, 7))</td>
<td>((-15, -21))</td>
</tr>
<tr>
<td>10. ((-3, 8))</td>
<td>((9, -24))</td>
</tr>
<tr>
<td>11. ((0, -2))</td>
<td>((0, 6))</td>
</tr>
<tr>
<td>12. ((6, 6))</td>
<td>((-18, -18))</td>
</tr>
</tbody>
</table>

Write a single rule for a dilation by which the image of \(A\) is \(A`\)

<table>
<thead>
<tr>
<th>Point</th>
<th>Rule</th>
<th>New Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>13. (A(2, 5))</td>
<td>(D_2)</td>
<td>(A`(4, 10))</td>
</tr>
<tr>
<td>14. (A(3, -1))</td>
<td>(D_7)</td>
<td>(A`(21, -7))</td>
</tr>
<tr>
<td>15. (A(10, 4))</td>
<td>(D_{1/2})</td>
<td>(A`(5, 2))</td>
</tr>
<tr>
<td>16. (A(-20, 8))</td>
<td>(D_{1/4})</td>
<td>(A`(-5, 2))</td>
</tr>
<tr>
<td>17. (A(4, 6))</td>
<td>(D_{3/2})</td>
<td>(A`(6, 9))</td>
</tr>
<tr>
<td>18. (A(4, -3))</td>
<td>(D_-2)</td>
<td>(A`(-8, 16))</td>
</tr>
<tr>
<td>19. (A(-2, 5))</td>
<td>(D_{-4})</td>
<td>(A`(8, -20))</td>
</tr>
<tr>
<td>20. (A(-12, 9))</td>
<td>(D_{2/3})</td>
<td>(A`(-8, 6))</td>
</tr>
<tr>
<td>21. (A(0, 9))</td>
<td>(D_{-1/3})</td>
<td>(A`(0, -3))</td>
</tr>
</tbody>
</table>
### NOTATION CONSISTENCY

#### REFLECTION

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{x \text{ axis}}$</td>
<td>Reflection over the x axis</td>
<td>$R_x$ is probably okay as well</td>
</tr>
<tr>
<td>$R_{y \text{ axis}}$</td>
<td>Reflection over the y axis</td>
<td>$R_y$ is probably okay as well</td>
</tr>
<tr>
<td>$R_{x=3}$</td>
<td>Reflection over the x = 3 line</td>
<td></td>
</tr>
<tr>
<td>$R_{y=x}$</td>
<td>Reflection over the y = 1x line</td>
<td></td>
</tr>
<tr>
<td>$R_m$</td>
<td>Reflection over line m</td>
<td></td>
</tr>
<tr>
<td>$R_{AB}$</td>
<td>Reflection over segment AB</td>
<td></td>
</tr>
<tr>
<td>$R_{AB}$</td>
<td>Reflection over line AB</td>
<td></td>
</tr>
</tbody>
</table>

A Reflection is recognizable because it will have only ONE item as a subscript... the line of reflection.

(Some use a small r for reflection and a capital R for rotation.)

#### ROTATION

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{O,89^\circ}$</td>
<td>Rotation about Point O for a positive 89°</td>
<td></td>
</tr>
<tr>
<td>When O is used it is implied that O = Origin at (0, 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{P,-134^\circ}$</td>
<td>Rotation about Point P for a negative 134°</td>
<td></td>
</tr>
<tr>
<td>$R_{(2,3),42^\circ}$</td>
<td>Rotation about location (2,3) for a positive 42°</td>
<td></td>
</tr>
</tbody>
</table>

A rotation is recognizable because it will have TWO items in the subscript... a center and a degree.

#### TRANSLATION

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{(-6,4)}$</td>
<td>Translate 6 left and 4 up</td>
<td></td>
</tr>
</tbody>
</table>

A translation is recognizable because it will have vector notation.

#### DILATION

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{O,3}$</td>
<td>Dilation from point O a scale factor of 3</td>
<td></td>
</tr>
<tr>
<td>$D_{O,\frac{1}{2}}$</td>
<td>Dilation from point O a scale factor of 1/2</td>
<td></td>
</tr>
<tr>
<td>$D_{A,-2}$</td>
<td>Dilation from point A a scale factor of -2</td>
<td></td>
</tr>
</tbody>
</table>
1. – 4. Which transformation has taken place?

1. **Rotation**

2. **Translation**

3. **Reflection**

4. **Rotation**

2. Complete the chart.

<table>
<thead>
<tr>
<th>Relationship between pre-image and image</th>
<th>ROTATION</th>
<th>REFLECTION</th>
<th>TRANSLATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distances</td>
<td>SAME OR DIFFERENT</td>
<td>SAME OR DIFFERENT</td>
<td>SAME OR DIFFERENT</td>
</tr>
<tr>
<td>Orientation</td>
<td>SAME OR DIFFERENT</td>
<td>SAME OR DIFFERENT</td>
<td>SAME OR DIFFERENT</td>
</tr>
</tbody>
</table>

3. Given that $\triangle ABC$ was mapped to $\triangle A'B'C'$ using a single transformation.
   a) Why couldn’t this mapping have resulted by a single translation?
      **Distance is not equal**
   b) What transformation must have mapped these two triangles?
      **Rotation**
      Explain your answer.
      a) orientation is the same
      b) Distance is different

4. Given that $\triangle ABC$ was mapped to $\triangle A'B'C'$ using a single transformation.
   a) Why couldn’t this mapping have resulted by a single reflection?
      Since the orientation is the same, it is not a reflection
      **Because orientation is the same**
   b) What transformation must have mapped these two triangles?
      **Rotation**
      Explain your answer.
      a) orientation is the same
      b) Distance is different
5. $\triangle ABC$ is congruent to $\triangle A'B'C'$. A student tries to determine which of these single transformations mapped $\triangle ABC$ onto $\triangle A'B'C'$. She concludes that a reflection had to be involved and more than one transformation had to map these on two triangles.

a) How can she conclude that a reflection was involved?  
Because orientation is different

b) How can she conclude that this wasn’t just a single reflection?  
If it is one reflection, $AA'$ would be parallel to $BB'$ and $CC'$

6. $BC'$ was translated by the arrow making $BC \equiv B'C'$ and $BC \parallel B'C'$.

a) What other segments in the diagram are congruent?  
$CC' \equiv BB'$

b) What other segments in the diagram are parallel?  
$CC' \parallel BB'$
1. **ROTATION BY 90° ABOUT THE ORIGIN (Use Patty Paper to help you with these!!)**

<table>
<thead>
<tr>
<th>PRE-IMAGE LOCATION</th>
<th>REFLECTION</th>
<th>IMAGE LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Place ΔDEF on the coordinate grid at D(0,4), E(3,4) and F(0,0)</td>
<td>$R_{O,90°}$</td>
<td>$D'(-4,0)$, $E'(-4,3)$, &amp; $F'(0,0)$</td>
</tr>
<tr>
<td>b) Place ΔABC on the coordinate grid at A(-3,6), B(-1,1) and C(-5,1)</td>
<td>$R_{O,90°}$</td>
<td>$A'(-6,-3)$, $B'(-1,-1)$, &amp; $C'(-1,-5)$</td>
</tr>
<tr>
<td>c) Place ΔMNP on the coordinate grid at M(-6,5), N(1,2) and P(-4,2)</td>
<td>$R_{O,90°}$</td>
<td>$M'(-5,-6)$, $N'(-2,1)$, &amp; $P'(-2,-4)$</td>
</tr>
</tbody>
</table>

Create a transformation rule for rotation about the origin, 90°. $R_{o,90°} = (-y, x)$

2. **ROTATION BY 180° ABOUT THE ORIGIN (Use Patty Paper to help you with these!!)**

<table>
<thead>
<tr>
<th>PRE-IMAGE LOCATION</th>
<th>REFLECTION</th>
<th>IMAGE LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Place ΔDEF on the coordinate grid at D(0,4), E(3,4) and F(0,0)</td>
<td>$R_{O,180°}$</td>
<td>$D'(0,-4)$, $E'(-3,-4)$, &amp; $F'(0,0)$</td>
</tr>
<tr>
<td>b) Place ΔABC on the coordinate grid at A(5,6), B(7,1) and C(3,1)</td>
<td>$R_{O,180°}$</td>
<td>$A'(-5,-6)$, $B'(-7,-1)$, &amp; $C'(-3,-1)$</td>
</tr>
<tr>
<td>c) Place ΔMNP on the coordinate grid at M(1,4), N(8,1) and P(3,1)</td>
<td>$R_{O,180°}$</td>
<td>$M'(-1,-4)$, $N'(-8,-1)$, &amp; $P'(-3,-1)$</td>
</tr>
</tbody>
</table>

Create a transformation rule for rotation about the origin, 180°. $R_{o,180°} = (-y, x)$

3. **ROTATION BY 270° (-90°) ABOUT THE ORIGIN (Use Patty Paper to help you with these!!)**

<table>
<thead>
<tr>
<th>PRE-IMAGE LOCATION</th>
<th>REFLECTION</th>
<th>IMAGE LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Place ΔDEF on the coordinate grid at D(0,4), E(3,4) and F(0,0)</td>
<td>$R_{O,270°}$</td>
<td>$D'(4,0)$, $E'(4,-3)$, &amp; $F'(0,0)$</td>
</tr>
<tr>
<td>b) Place ΔABC on the coordinate grid at A(-3,6), B(-1,1) and C(-5,1)</td>
<td>$R_{O,270°}$</td>
<td>$A'(6,3)$, $B'(1,1)$, &amp; $C'(1,5)$</td>
</tr>
<tr>
<td>c) Place ΔMNP on the coordinate grid at M(-3,5), N(-6,-2) and P(-6,3)</td>
<td>$R_{O,270°}$</td>
<td>$M'(5,3)$, $N'(-2,6)$, &amp; $P'(3,6)$</td>
</tr>
</tbody>
</table>

Create a transformation rule for rotation about the origin, 270°. $R_{o,270°} = (-y, x)$

4. Using your rules above, determine the location of the missing location.

a) $R_{o,90°}(-6,4) = (-4,-6)$

b) $R_{o,180°}(-3,-5) = (3,-5)$
c) $R_{o, 270} \begin{bmatrix} 4, -3 \end{bmatrix} = \begin{bmatrix} -3, -4 \end{bmatrix}$
d) $R_{o, 90} \begin{bmatrix} 7, -8 \end{bmatrix} = \begin{bmatrix} 7, 8 \end{bmatrix}$
e) $R_{y-axis} \begin{bmatrix} -8, 4 \end{bmatrix} = \begin{bmatrix} 8, 4 \end{bmatrix}$
f) $R_{o, -270} \begin{bmatrix} -5, -8 \end{bmatrix} = \begin{bmatrix} 8, -5 \end{bmatrix}$
g) $R_y \begin{bmatrix} 2, 8 \end{bmatrix} = \begin{bmatrix} 2, -8 \end{bmatrix}$
h) $R_{o, 180} \begin{bmatrix} 0, -9 \end{bmatrix} = \begin{bmatrix} 0, 9 \end{bmatrix}$
i) $R_{x-axis} \begin{bmatrix} 3, 5 \end{bmatrix} = \begin{bmatrix} 3, -5 \end{bmatrix}$
j) $R_{o, 180} \begin{bmatrix} -4, 2 \end{bmatrix} = \begin{bmatrix} 4, -2 \end{bmatrix}$
k) $R_{y-axis} \begin{bmatrix} 4, 2 \end{bmatrix} = \begin{bmatrix} -4, 2 \end{bmatrix}$
l) $R_{y-axis} \begin{bmatrix} -2, 15 \end{bmatrix} = \begin{bmatrix} 2, 15 \end{bmatrix}$
m) $R_{o, 90} \begin{bmatrix} -3, -9 \end{bmatrix} = \begin{bmatrix} 9, -3 \end{bmatrix}$
n) $R_{o, -90} \begin{bmatrix} 10, 3 \end{bmatrix} = \begin{bmatrix} 3, 10 \end{bmatrix}$

1. – 6. Use the grid to rotate the following figures. Label the image.

7. – 9. Circle the center of rotation for the following pre-image and images.
7. A rotation of $90^\circ$

8. A rotation of $180^\circ$

9. A rotation of $180^\circ$

10. Determine the pre-image coordinates, then rotate it, and determine the image coordinates.

   \[ R_{O,90^\circ}(\Delta ABC) \]
   
   a) \( A = (2, 6) \) \quad A' = (-6, 2)
   
   b) \( B = (5, 6) \) \quad B' = (-6, 5)
   
   c) \( C = (1, 1) \) \quad C' = (-1, 1)

   \[ R_{O,90^\circ}(\Delta DFE) \]
   
   d) \( D = (-7, 2) \) \quad D' = (2, 7)
   
   e) \( E = (-2, -3) \) \quad E' = (3, -2)
   
   f) \( F = (-2, -7) \) \quad F' = (7, -2)

11. Determine the name of the point that meets the given conditions.

   a) \( R_{G,60^\circ}(A) = B \)
   
   b) \( R_{G,180^\circ}(B) = E \)
   
   c) \( R_{G,300^\circ}(D) = C \)
   
   d) \( R_{G,-120^\circ}(D) = B \)
   
   e) \( R_{G,240^\circ}(E) = C \)
   
   f) \( R_{G,-240^\circ}(F) = B \)
   
   g) \( R_{A,60^\circ}(B) = G \)
   
   h) \( R_{C,120^\circ}(D) = B \)